8.7 Complete the Square

Steps for Completing the Square

Step 1: Move the constant "c" to the other side of the equation.
Step 2: If there is a number in front of the x^2 term, then divide EVERYTHING by that number.
Step 3: Divide the "b" term by 2, and then square it.
Step 4: Add that new number to both sides of the equation.
Step 5: Shrink the left side of the equation to $(x + \frac{b}{2})^2$
Step 6: Square root both sides to get rid of the square. Remember to put a $\pm$ with the number on the right side.
Step 7: Get x by itself.

Finding a Perfect Square Trinomial
What number completes the square in each of the following?

a) $x^2 - x + \underline{1/4}$ $(x - \frac{1}{2})(x - 1/2)$

$(\frac{-1}{2})^2 = \frac{1}{4}$

b) $x^2 + 20x + \underline{100}$ $(x+10)(x+10)$

$(\frac{20}{2})^2 = 100$

c) $x^2 + 4x + \underline{4}$ $(x+2)(x+2)$

$(\frac{4}{2})^2 = 4$

Solving Quadratic Equations by Completing the Square
Solve each of the following quadratic equations by completing the square.

a) $x^2 + 4x - 12 = 0$

$(\frac{4}{2})^2 = 4$

$x^2 + 4x + \underline{4} = 12 + \underline{4}$

$(x+2)(x+2) = 16$

$(x+2)^2 = 16$

$x + 2 = \pm 4$

$x + 2 = 4 \qquad x + 2 = -4$

$x = 2 \qquad x = -6$

b) $x^2 - 12x + 7 = 0$

$(\frac{12}{2})^2 = 36$

$x^2 - 12x + \underline{36} = -7 + \underline{36}$

$(x-6)(x-6) = 29$

$(x-6)^2 = 29$

$x - 6 = \pm\sqrt{29}$

$x - 6 = \sqrt{29} \qquad x - 6 = -\sqrt{29}$

$x = 6 + \sqrt{29} \qquad x = 6 - \sqrt{29}$

c) $x^2 + 2x + 3 = 0$

$x^2 + 2x + \underline{1} = -3 + \underline{1}$

$(x+1)(x+1) = -2$

$(x+1)^2 = -2$

$x + 1 = \pm\sqrt{-2}$

$x + 1 = \pm i\sqrt{2} \qquad x + 1 = -i\sqrt{2}$

$x = -1 + i\sqrt{2} \qquad x = -1 - i\sqrt{2}$

d) $\frac{3x^2}{3} + \frac{15x}{3} - \frac{21}{3} = \frac{0}{3}$

$x^2 + 5x - 7 = 0$

$x^2 + 5x + \underline{6.25} = +7 + \underline{6.25}$

$(x + 5/2)(x + 5/2) = 53/4$

$(x + 5/2)^2 = 53/4$

$x + 5/2 = \pm\sqrt{\frac{53}{4}}$

$x + 5/2 = +\frac{\sqrt{53}}{2} \qquad x + 5/2 = -\frac{\sqrt{53}}{2}$

$x = \frac{-5 + \sqrt{53}}{2} \qquad x = \frac{-5 - \sqrt{53}}{2}$

In-Class Practice: Solve the following quadratic equations by completing the square.

1. Solve for x: $x^2 - 10x + 26 = 8$

$x^2 - 10x + \underline{25} = -26 + 8 + \underline{25}$

$(x-5)(x-5) = 7$

$(x-5)^2 = 7$

$x - 5 = \pm\sqrt{7}$

$x = 5 \pm \sqrt{7}$

2. Solve for k: $k^2 - 4k + 1 = -5$

$x^2 - 4x + \underline{4} = -1 - 5 + \underline{4}$

$(x-2)(x-2) = -2$

$(x-2)^2 = -2$

$x - 2 = \pm i\sqrt{2}$

$x = 2 \pm i\sqrt{2}$

3. Solve for x: $x^2 + 14x - 15 = 0$

$x^2 + 14x + \underline{49} = 15 + \underline{49}$

$(x+7)(x+7) = 64$

$(x+7)^2 = 64$

$x + 7 = \pm 8$

$x + 7 = 8 \quad x + 7 = -8$

$x = 1 \quad x = -15$

4. Solve for k: $k^2 - 12k + 23 = 0$

$x^2 - 12x + \underline{36} = -23 + \underline{36}$

$(x-6)(x-6) = 13$

$(x-6)^2 = 13$

$(x-6) = \pm\sqrt{13}$

$x = 6 \pm \sqrt{13}$

Solve the following equation using each method below: $a^2 + 14a = 51$

Factoring:	Quadratic Formula:	Complete the Square:
$a^2 + 14a - 51 = 0$ -51, 17, -3, 14 $(a+17)(a-3) = 0$ $a+17=0 \quad a-3=0$ $a=-17 \quad a=3$	$a^2 + 14a - 51 = 0$ $x = \frac{-14 \pm \sqrt{(14)^2 - 4(1)(-51)}}{2(1)}$ $x = \frac{-14 \pm \sqrt{400}}{2}$ $x = \frac{-14 \pm 20}{2}$ $x = \frac{-14+20}{2} = \frac{6}{2} = 3$ $x = \frac{-14-20}{2} = \frac{-34}{2} = -17$	$a^2 + 14a + \underline{49} = 51 + \underline{49}$ $(a+7)(a+7) = 100$ $(a+7)^2 = 100$ $a+7 = \pm 10$ $a+7=10 \quad a+7=-10$ $a=3 \quad a=-17$